

Flower Pollination Algorithm (FPA) to Solve Quadratic Assignment Problem (QAP)

Derby Prayogo Samdean¹, Herry Suprajitno^{2,*}, Edi Winarko³

^{1,2,3}Department of Mathematics, Faculty of Science and Technology, Universitas Airlangga

*Corresponding author: herry-s@fst.unair.ac.id

Abstract. The purpose of this paper is to solve Quadratic Assignment Problem using Flower Pollination Algorithm. Quadratic Assignment Problem discuss about assignment of facilities to locations in order to minimize the total assignment costs where each facility assigns only to one location and each location is assigned by only one facility. Flower pollination Algorithm is an algorithm inspired by the process of flower pollination. There are two main steps in this algorithm, global pollination and local pollination controlled by switch probability. The program was created using Java programming language and implemented into three cases based on its size: small, medium and large. The computation process obtained the objective function value for each data using various values of parameter. According to the pattern of the computational result, it can be concluded that a high value of maximum iteration of the algorithm can help to gain better solution for this problem.

Keyword: *Quadratic Assignment Problem, Flower Pollination Algorithm.*

1 Introduction

In the modern era, the development of the industrial sector is increasing. According to Kartasapoetra [1], the definition of industry is an economic activity processing raw materials, semi-finished goods or finished goods into goods a higher value of use, including industrial design activities and industrial engineering. Along with the development of technology, industrial companies must continue to innovate which aims to optimize all processes and needs in the industry. One effort to optimize a company is to have effective management. According to Singh [2], many ways that companies can use to optimize those process. One that has a significant impact on the effectiveness and efficiency of a company is planning assignments. The optimization of the assignment causes the company to obtain optimal results with minimum expenditure.

The assignment problem is an assignment of work to the existing resources. Each resource (officer, facility, machine) is specifically assigned to an activity or task. The main goal is to minimize the total cost or time needed to carry out existing tasks [3]. According to Herjanto [4], layout planning is one stage in planning a facility that aims to develop an efficient and effective production system so that it can achieve a production process at the most economical cost. However, in direct application in the industrial world, there are

still a number of companies that design facility layouts based on previous company experience or based on the order of the process of using facilities at the closest distance. In fact, there are still companies that do not pay attention to the facilities layout so that it can disrupt the performance of the employees. Therefore there is needed optimization in the layout of company facilities so that we can optimize the processes that exist in the company.

According to Tate and Smith [5], Quadratic Assignment Problem (QAP) is an assignment problem whose objective function is quadratic and is one of the principal combinatorial optimization problems. QAP consists of searching for the best placement of n facilities to n locations, with the known of flow between facilities and the distance between locations, it will minimize the objective function expressed from the cost (distance between locations and flows between facilities) associated with the assignment pair [6]. In previous studies, several algorithms have been used to solve QAP. Some of them are Genetic Algorithm [7], Simulated Annealing [8], Tabu Search [9], and Ant Colony Optimization [10].

The Flower Pollination Algorithm, known as Flower Pollination Algorithm (FPA), is an algorithm developed by Xin-She Yang in 2012. FPA is an algorithm inspired by the flower pollination process. The main purpose of the flower is to reproduce by pollination. Based on pollinators, pollination can be divided into two types, biotic and abiotic. Biotic pollination is pollination aided by organisms while abiotic pollination is pollination without the involvement of organisms. Pollination can be done in two ways, self-pollination can be interpreted as pollination between flowers in one tree. Cross pollination can be interpreted as pollination whose pollens come from flowers belonging to another tree. The transfer of the pollens is often associated with pollinators, such as insects, birds, and other animals. In fact, some flowers have connections that are so close to their pollinators. Flowers that have pollinators still have the advantage that the rate of transfer of pollen to similar flowers is more maximal. Likewise with pollinators who have the advantage of being able to ensure the presence of nectar in similar flowers with minimum learning. This can be considered as a process of optimization of plants. All factors and processes of pollinating flowers to optimize reproduction of plants [11].

According to Yang [11], there are four rules in Flower Pollination Algorithm (FPA). First, biotic and self-pollination can be considered processes of global pollination, and pollen-carrying pollinators move that obeys *Lévy flights*. Second, for local pollination, abiotic pollination and self-pollination are used. Third, Pollinators such as insects can develop flower constancy, which is equivalent to a reproduction probability that is proportional to the similarity of two flower involved. Fourth, The interaction or switching of local pollination and global pollination can be controlled by a switch probability $p \in [0,1]$, slightly biased toward local pollination.

Flower Pollination Algorithm has superiority in the form better results compared to Genetic Algorithm (GA), Particle Swarm Optimization (PSO) with the same number of iterations because it uses Lévy flight to implement random walks from insect pollinators to get more optimal results [11]. Based on the explanation that has been explained, it is very interesting to apply Flower Pollination Algorithm (FPA) to solve Quadratic Assignment Problem (QAP).

2 Quadratic Assignment Problem (QAP)

Quadratic assignment problem is an assignment problem whose objective function is quadratic. QAP generally has a concept similar to the assignment of facilities at a location with the aim of minimizing the cost of material movement between facilities. The objective function with a quadratic form involves multiplying two independent variables. Quadratic forms are widely used in objective functions for facility layout problems [12].

In general, the quadratic assignment model is as follows [13]:

Minimize:

$$Z = \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{l=1}^n f_{ik} d_{jl} x_{ij} x_{kl} \quad (1)$$

And constraints:

$$\sum_{j=1}^n x_{ij} = 1, \quad i = 1, 2, \dots, n \quad (2)$$

$$\sum_{i=1}^n x_{ij} = 1, \quad j = 1, 2, \dots, n \quad (3)$$

$$x_{ij} \in \{0, 1\}, \quad i, j = 1, 2, \dots, n$$

where :

Z : total cost to assign n facilities to n locations.

f_{ik} : flow from facility i to facility k .

d_{jl} : distance between location j and location l .

x_{ij} : assign facility i to location j , with

$$x_{ij} = \begin{cases} 1, & \text{if facility } i \text{ is assign to location } j \\ 0, & \text{others} \end{cases} ; i, j = 1, 2, \dots, n$$

x_{kl} : assign facility k to location l , where

$$x_{kl} = \begin{cases} 1, & \text{if facility } k \text{ is assign to location } l \\ 0, & \text{others} \end{cases} ; k, l = 1, 2, \dots, n$$

n : number of facilities

3 Flower Pollination Algorithm (FPA)

An algorithm is a set of finite rules that provide a series of operations to solve a particular type of problem [14]. Flower Pollination Algorithm (FPA) is one of the optimization algorithms that can be used for decision making. This algorithm mimics the behavior of pollination of flowers by insects that exist in the universe. Insects will move from flower to other flowers to help pollinate flowers. Of course, flowers that look better will be chosen by an insect to be infested than flowers that look less good. FPA was developed by Xin-She in 2012 [11]. In general, there are four rules that are used:

1. Biotic and self-pollination can be considered processes of global pollination, and pollen-carrying pollinators move that obeys *Lévy flights*.
2. For local pollination, abiotic pollination and self-pollination are used.
3. Pollinators such as insects can develop flower constancy, which is equivalent to a reproduction probability that is proportional to the similarity of two flowers involved.
4. The interaction or switching of local pollination and global pollination can be controlled by a switch probability $p \in [0,1]$, slightly biased toward local pollination.

Broadly speaking, this algorithm is divided into two processes [11], there are:

a. Global Pollination

Rules one and three can be presented as mathematical model

$$x_i^{t+1} = x_i^t + \gamma L(g_* - x_i^t) \quad (4)$$

x_i^t is pollen i or solution vector x_i at iteration t , g_* is the best solution between all solutions at iteration t , and γ is scaling factor to control step size. L is *Lévy flight* drawn from *Lévy distribution*.

$$L \sim \frac{\lambda \Gamma(\lambda) \sin\left(\frac{\pi\lambda}{2}\right)}{\pi} \frac{1}{s^{1+\lambda}} \quad (5)$$

where Γ is gamma function and λ is parameter for gamma function.

Step length (s) can be calculated by:

$$s = \frac{U}{|V|^\lambda} \quad (6)$$

V is a random real number that are drawn from a Gaussian normal distribution with a zero mean and variance of 1, and U is a random real number that are drawn from a Gaussian normal distribution with a zero mean and variance can be calculated by:

$$\sigma^2 = \left[\frac{\Gamma(1 + \lambda)}{\lambda \Gamma\left(\frac{1 + \lambda}{2}\right)} \cdot \frac{\sin\left(\frac{\pi\lambda}{2}\right)}{2^{\frac{\lambda-1}{2}}} \right]^{\frac{1}{\lambda}} \quad (7)$$

b. Local Pollination

Rules two can be presented as mathematical model

$$x_i^{t+1} = x_i^t + \epsilon(x_j^t - x_k^t) \quad (8)$$

x_j^t and x_k^t are two different flower at iteration t . where ϵ is a random real number that are drawn from a uniform distribution in $[0,1]$.

According to Diethelm [16], function $\Gamma: (0, \infty) \rightarrow \mathbb{R}$, is defined as

$$\Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x} dx \quad (9)$$

is called Gamma function and satisfy $\Gamma(n+1) = n\Gamma(n)$, where $n \in \mathbb{R}^+$ and $\Gamma(1-n)\Gamma(n) = \frac{\pi}{\sin \pi n}$, where $n \in \mathbb{R}^-$

4 Procedure to solve Quadratic Assignment Problem (QAP) with Flower Pollination Algorithm (FPA)

The steps used in solving the problems in this study are as follows:

1. Input the number of facilities, distance between locations, and flow between facilities.
2. Input parameter of flower pollination algorithm there are number of flower (n), maximum iteration, stepsize control (γ), switch probability p , and value of λ .
3. Generate initial population of flower
 - i. Generates random real numbers as many as existing facilities.
 - ii. Sort random real numbers so that the sequence numbers of random numbers are obtained.
 - iii. Shows the sequence number according to the initial random number.
 - iv. Determine the placement of facilities to existing locations.
4. Calculate total cost based on flow between facilities and distance between locations that will be the objective function (F_i).
5. Find temporary best solution (g_*) from all flower by sorting the value of the objective function from the smallest to the largest.
6. Generate real random number $(0,1) p_i$.
 - if $p_i < p$ do global pollination with following steps :
 - i. Calculate value σ^2 .
 - ii. Get value U by generating real random number drawn from normal distribution $N(0, \sigma^2)$. Get value V by generating real random number drawn from normal distribution $N(0,1)$.
 - iii. Calculate s by dividing each element of U with each element of V positively to the power of $\frac{1}{\lambda}$.
 - iv. Calculate L .

- v. Get new flower (x_i) by multiplying L and γ with result of subtraction of best solution value at current iteration (g_*) with current flower value, then added by current flower value.
- if $p_i \geq p$ do local pollination with following steps :
 - i. Generate value ϵ by generating real random number drawn from normal distribution $U(0,1)$.
 - ii. Get new flower (x_i) by multiplying ϵ with the result of subtraction of value from two random flower at current population, then added by current flower value.
- 7. Calculate value of new flower objective function.
- 8. Compare the value of new objective function with value of old objective function. If the value of new objective function is better, then replace the value of old objective function with the value of new objective function and save the solution. Else, use the value of old objective function.
- 9. Determine the best solution (g_*) at current population.
- 10. Repeat step 6 to 9 until maximum iteration reached.
- 11. Display the best solution (g_*).

5 Result and Discussion

The data used in study are taken from Dorigo [16], Hadley [17], and Krarup [18]. The data used in this study are small data with 4 facilities 4 locations [16], medium data with 18 facilities 18 locations [17], and large data with 32 facilities 32 locations [18]. The following is a comparison of the best solutions for small data, medium data, and large data using variations in the parameters of the amount of flower, maximum iteration, switch probability and value $\alpha=0.01$ [19], $\beta=1.5 \in [1,2]$ [20]. This solution is obtained using Java programming language with Netbeans 8.2. The result of running program for small data are presented in Table 1, medium data are presented in Table 2, and large data presented in Table 3.

Table 1 Result of Running Program for Small Data

Maximum iteration	Number of Flower	Switch Probability		
		0.1	0.5	0.9
10	10	1340	1340	1340
	100	1340	1340	1340

	1000	1340	1340	1340
100	10	1340	1340	1340
	100	1340	1340	1340
	1000	1340	1340	1340
1000	10	1340	1340	1340
	100	1340	1340	1340
	1000	1340	1340	1340

In Tabel 1, it can be seen that with different parameter values it obtain the same solution with total cost is 1340 unit costs. This is due to the small number of facilities so that the order changes in facilities are limited. These results are optimal solutions based on Dorigo's research [17] with solutions to problems 3 – 1 – 2 – 4, which means facility 3 is placed in location 1, facility 1 is placed in location 2, facility 2 is placed at location 3, and facility 4 is placed at location 4.

Table 2 presents computational result of medium data. The best objective value is 5356 unit cost with parameter amount of flower = 100, switch probability = 0.9, and maximum iteration = 1000. The order of placement facilities obtained is 8 – 15 – 1 – 6 – 7 – 18 – 14 – 11 – 16 – 10 – 12 – 5 – 3 – 13 – 2 – 17 – 9 – 4. From Table 2 shows that the greater amount of flower and maximum iteration, the objective function will be better, while variations in switch probability parameters do not necessarily obtain the best objective function.

From Tabel 3, the best objective value is 97220 unit cost with parameter amount of flower = 250, switch probability = 0.1, and maximum iteration = 1000. The order of placement facilities obtained is 15 – 26 – 32 – 18 – 22 – 6 – 4 – 27 – 3 – 2 – 5 – 16 – 12 – 1 – 17 – 11 – 30 – 24 – 25 – 31 – 28 – 29 – 21 – 14 – 19 – 23 – 20 – 13 – 10 – 8 – 7 – 9. Table 3 shows that the greater amount of maximum iteration, the objective function will be better, while variations in switch probability parameters and amount of flower do not necessarily obtain the best objective function.

Table 2 Result of Running Program for Medium Data

Maximum Iteration	Amount of Flower	<i>Switch Probability</i>		
		0.1	0.5	0.9
10	10	5659	5755	5706
	100	5547	5611	5511
	1000	5537	5548	5474
100	10	5640	5609	5503
	100	5456	5531	5438
	1000	5406	5399	5381
1000	10	5516	5438	5419
	100	5412	5394	5368
	1000	5370	5391	5356

Table 3 Result of Running Program for Large Data

Maximum Iteration	Amount of Flower	<i>Switch Probability</i>		
		0.1	0.5	0.9
10	10	124200	119960	116120
	100	120850	121640	122560
	250	114580	118190	115730
100	10	114420	114610	114280
	100	107290	114190	109050
	250	108280	108970	106950
1000	10	108880	107750	113530
	100	100840	100150	100920
	250	97220	104430	105590

6 Conclusion

Flower Pollination Algorithm (FPA) can be applied to solve Quadratic Assignment Problem (QAP). From the three data used, it can be seen that the change in the maximum iteration parameter affects the value of the objective function, but the change in the value of the switch probability parameter and the amount of interest does not significantly influence. The more the maximum number of iterations, the better solution will be obtained.

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