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Mathematical Modelling of Household Plastic Waste Distribution Management with Transportation Method

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Abstract. Most of the plastic waste is generated by households. Plastic waste is often a problem because there are many and cannot decompose by itself. To solve the problem of plastic waste, plastic waste management will be carried out by channeling plastic waste to the waste bank. Good and correct management of plastic waste distribution is carried out by applying transportation methods starting from modeling the distribution of plastic waste generated from households and solutions from mathematical modeling that is built. From the transportation calculation, the minimum cost of managing the distribution of plastic waste will be obtained so that an efficient plastic waste distribution management system is formed. This study uses the type of experimental research (experimental research) with a literature study. The data used to construct the model are derived from literature studies and simulation data. The results of this study can be developed into a computer program regarding the management of household plastic waste distribution.

Keywords: *plastic waste; Mathematics Modelling, Transportation Method.*

1 Introduction

According to Law No. 18 of 2008 on waste management, waste is the rest of human daily activities and or the rest of natural processes in the form of solids. Waste is also defined as the remnants of materials that undergo a treatment, either because it has been taken the main part, has been through the process of processing or has not provided socio-economic benefits and can cause pollution or disruption to the environment [1]. Based on Law No. 18 of 2008 it is also stated that one type of waste managed is household waste [2].Household waste is garbage that comes from daily activities in the household, but does not include feces and specific waste. Similar waste that is also household waste is garbage that comes from commercial areas, industrial areas, special areas, social facilities, public facilities, and or other facilities. Meanwhile, the specific waste here is interpreted asgarbage that contains hazardous and toxic materials, garbage that contains hazardous and toxic material waste, garbage arising from disasters, building debris, garbage that is technologically un-processable, and garbage that arises non-periodically [3].

In household waste, most of what is produced is plastic waste. This plastic waste is often a problem because of the large amount and cannot decompose by itself. To reducethe buildup of plastic waste, most people choose to burn, even though this action actually causes more problems, such as air pollution [4]. Therefore, it is necessary to do good and correct plastic waste management. In this case there is a solution to manage plastic waste in its place, namely through a garbage bank with the help of janitors as couriers who will take plastic waste in people's homes [5]. Meanwhile, in its application the janitor is unlikely to come every day to pick up the plastic waste. This is due to the large number of people's homes that must be visited and the acquisition of too little plastic waste when taken every day so it is considered less efficient. Solutions so that plastic waste management can run efficiently, it is necessary to create a mathematical model of managing the distribution of plastic waste produced from households. From this model willthen be seen how the distribution process is efficient so that the management of household plastic waste will not only overcome the problem of household waste buildup but also effective in the transportation of waste carried out by janitors.

For the realization of an efficient solution, the mathematical model that has been formed next will be determined by using transportation methods in the operations research lecture. The process that will be carried out will later become an efficient plastic waste distribution management system so that it can be used by the community. In general, the management of household plastic waste distribution starts from households (consumers) who have collected plastic waste according to predetermined targets, then will be transported by officers or we can call drivers (couriers) to the garbage bank. In determiningthe route of transporting plastic waste, of course, must use an efficient route so that its distribution is efficient. To find an efficient route can be calculated by method of transportation. From the calculation of transportation methods obtained a solution to carry out the efficient transportation of plastic waste. Thus, the management of household plasticwaste distribution with transportation methods can produce an efficient plastic waste distribution system.

2 Method

In this study, the application of transportation methods was used to bring up mathematical modeling of the management of plastic waste distribution as well as the solutions obtained from the modeling. This solution is in the form of the minimum cost of managing the distribution of household plastic waste.

2.1 Transportation Method

In general, transportation problems relate to the distribution of a single product from multiple sources, with limited supply, to multiple destinations, with a certain demand, at a minimum transport cost because there is only one kind of goods, a destination can meet its demand from one or more sources [6]. If it is brought into the problem of garbage then the community house is declared as the place of origin and the intended place is declared as a garbage bank. Special characteristics of the use of transportation methods, especially for garbage, are as follows.

1. There are a number of sources and a certain number of purposes.
2. Quantity of waste attributable from each source and requested by each specific major purpose.
3. Waste transported from a source to a large destination in accordance with the demand or capacity of the source.
4. The cost of transporting waste from a source to a specific major destination.

2.2 Mathematical Modeling

Mathematically, transportation problems can be modeled mathematically as follows.
Purpose function:

$$Min\ z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} X_{ij} \quad (1)$$

With constraints:

$$\sum_{i=1}^{n} X_{ij} = s_i \quad for\ i = 1, 2, \dots, m \quad (2)$$

$$\sum_{i=1}^{n} X_{ij} = d_i \quad for\ i = 1, 2, \dots, n \quad (3)$$

$$X_{ij} \geq 0 \quad for\ all\ i \text{ and } j \quad (4)$$

Information:

Z = Total cost of transportation
C_{ij} = transportation costs per unit of goods from source *i* to destination *j*
X_{ij} = number of goods distributed from source *i* to destination *j*
s_i = number of items available from source *i*
d_j = number of requests for goods at the destination *j*
m = many sources
n = many destination

Transportation problems can be placed in a special table called a transportation table. Sources are written in rows and destinations in columns. In the transportation table there is *m* x *n* box. Transportation costs per unit of goods C_{ij} are recorded in a small boxat the top right of each box. Requests from each destination have a rightmost column. The bottom left corner box indicates the fact that supply (*S*) is the same as demand or demand (*D*). The variable X_{ij} on each box indicates the number of goods transported from source *i* to destination *j*. A transportation problem is said to be balanced (balance program) if the amount of supply at the source *i* equals the amount of demand at the destination [7].

2.2.1 Vogel Approximation Method (VAM)

One method to find the initial feasible solution of a transportation problem with the following steps:

1. Create a transportation table.
2. Calculate the penalty for each row and column by subtracting the two lowest costs from each row and column.
3. Select the column / row with the largest penalty (if there is the same penalty, it can be selected one of them).
4. Allocate as many products (adjusted for capacity and demand) in the cells that have the lowest cost in the rows/columns that have the largest penalty.
5. Fully filled rows/columns cannot be re-entered in the process of calculating the next penalty search.
6. Do it again in step 2 until all products are allocated

2.2.2 Modified Distribution Method (MODI).

After getting the initial feasible solution, the next optimal solution is sought using the MODI method. The steps for the Modi method are as follows:

1. Fill the first table.
2. Determine the values of rows and columns. The first line is always given a zero value. Line value $\boldsymbol{W = Rw = 0}$. The value of the other rows and the values of all columns are determined based on the equation.
3. Calculate the repair index. The repair index is the value of an empty box.
4. Choose the starting point of change. A box that has a negative repair index means that when given an allocation will reduce the amount of transportation costs. When the value is positive it means charging will lead to an increase in the cost of carriage. The box that is the starting point of change is a box whose index is marked negative and the number is large.
5. Improve allocation
6. Repeat the steps starting from step 2 until the lowest cost is obtained, i.e. when there is no longer a negative index.

3 Results and Discussion

3.1 Samples and Data

In this study used samples and data from Gayungan subdistrict. Data taken in the formof the number of RW (in this case households are jumped into RW), the garbage bank is active along with its capacity as in Table 1.

Table 1 Waste Bank Capacity

Name of Waste Bank	Capacity
Ketintang 17 (A)	8.200 Kg
Seruni (B)	8.500 Kg
GKS (C)	8.601

Then data on the amount of waste generated in each RW (every month)

Dukuh Menanggal :

RW 1, RW5, RW3= 563,4 kg, RW2, RW4 = 565,2 kg, dan RW6, RW7, RW8, RW9 = 561,6 kg

Ketintang :

RW1, RW 2 = 1.132,1 kg dan RW 3, RW 4, RW5 , RW 6, RW 7, RW 8 = 1.130,4 kg

Menanggal :

RW1= 943,2 kg, RW2, RW3 = 945 kg, dan RW4, RW5, RW6 = 941,4 kg

Gayungan :

RW1, RW3 = 982,8 kg, RW2 = 984,6 kg, dan RW4, RW5, RW 6, RW7= 981 kg

In addition, transportation cost data from each RW to each Waste Bank as in Table 2, Table3, Table 4 and Table 5.

Table 2 The Transport Costs in Gayungan Village (in Thousands)

Waste Bank	RW					
	1	2	3	4	5	6
A	11	8,8	8,8	8,8	10	9,8
B	14	11	10	14	13	12
C	8,8	8,8	8,8	8,8	9,1	8,8

Table 3 The Transport Costs in Menanggal Village (in Thousands)

Waste Bank	RW						
	1	2	3	4	5	6	7
A	8,8	8,8	8,8	9,1	8,8	8,8	9,2
B	9,4	8,8	8,8	11	11	9,6	12
C	8,8	8,8	9	8,8	8,8	8,8	8,8

Table 4 The Transport Costs in Ketintang Village (in Thousands)

Waste Bank	RW							
	1	2	3	4	5	6	7	8
A	8,8	8,8	5	8,8	8,8	8,8	8,9	8,8
B	8,8	8,8	5	8,8	9,2	8,8	8,8	8,8
C	9	8,8	8,8	10	5	9	8,9	8,8

Table 5 The Transport Costs in Dukuh Menanggal Village (in Thousands)

Waste	RW								
	1	2	3	4	5	6	7	8	9

A	14	16	14	14	14	13	12	13	13
B	14	16	17	14	14	13	12	13	13
C	11	11	11	12	11	11	11	11	12

3.2 Mathematical Modeling

The formulation of the Linear Program Model for this problem is as follows:
Minimize :

$$Z = 14\ X_{1A} + 14X_{1B} + 11X_{1C} + 16\ X_{2A} + 16X_{2B} + 11X_{2C} + 14\ X_{3A} + 17X_{3B} + 11X_{3C} + 14\ X_{4A} + 14X_{4B} + 12X_{4C} + 14\ X_{5A} + 14X_{5B} + 11X_{5C} + 13X_{6A} + 13X_{6B} + 11X_{6C} + 12X_{7A} + 12X_{7B} + 11X_{7C} + 13X_{8A} + 13X_{8B} + 11X_{8C} + 13X_{9A} + 13X_{9B} + 12X_{9C} + 8{,}8X_{10A} + 8{,}8X_{10B} + 9X_{10C} + 8{,}8X_{11A} + 8{,}8X_{11B} + 8{,}8X_{11C} + 5X_{12A} + 5X_{12B} + 8{,}8X_{12C} + 8{,}8X_{13A} + 8{,}8X_{13B} + 10X_{13C} + 8{,}8X_{14A} + 9{,}2X_{14B} + 5X_{14C} + 8{,}8X_{15A} + 8{,}8X_{15B} + 9X_{15C} + 8{,}9X_{16A} + 8{,}8X_{16B} + 8{,}9X_{16C} + 8{,}8X_{17A} + 8{,}8X_{17B} + 8{,}8X_{17C} + 11X_{18A} + 14X_{18B} + 8{,}8X_{18C} + 8{,}8X_{19A} + 11X_{19B} + 8{,}8X_{19C} + 8{,}8X_{20A} + 10X_{20B} + 8{,}8X_{20C} + 8{,}8X_{21A} + 14X_{21B} + 8{,}8X_{21C} + 10X_{22A} + 13X_{22B} + 8{,}8X_{22C} + 9{,}8X_{23A} + 12X_{23B} + 8{,}8X_{23C} + 8{,}8X_{24A} + 9{,}4X_{24B} + 8{,}8X_{24C} + 8{,}8X_{25A} + 8{,}8X_{25B} + 8{,}8X_{25C} + 8{,}8X_{26A} + 8{,}8X_{26B} + 9X_{26C} + 9{,}1X_{27A} + 11X_{27B} + 8{,}8X_{27C} + 8{,}8X_{28A} + 11X_{28B} + 8{,}8X_{28C} + 8{,}8X_{29A} + 9{,}6X_{29B} + 8{,}8X_{29C} + 9{,}2X_{30A} + 12X_{30B} + 8{,}8X_{30C}$$

With Restrictions:

$$X_{1A} + X_{1B} + X_{1C} = 563{,}4$$
$$X_{2A} + X_{2B} + X_{2C} = 565{,}2$$
$$X_{3A} + X_{3B} + X_{3C} = 563{,}4$$
$$X_{4A} + X_{4B} + X_{4C} = 565{,}4$$
$$X_{5A} + X_{5B} + X_{5C} = 563{,}4$$
$$X_{6A} + X_{6B} + X_{6C} = 561{,}6$$
$$X_{7A} + X_{7B} + X_{7C} = 561{,}6$$
$$X_{8A} + X_{8B} + X_{8C} = 561{,}6$$
$$X_{9A} + X_{9B} + X_{9C} = 561{,}6$$
$$X_{10A} + X_{10B} + X_{10C} = 1.132{,}1$$
$$X_{11A} + X_{11B} + X_{11C} = 1.132{,}1$$
$$X_{12A} + X_{12B} + X_{12C} = 1.130{,}4$$
$$X_{13A} + X_{13B} + X_{13C} = 1.130{,}4$$
$$X_{14A} + X_{14B} + X_{14C} = 1.130{,}4$$
$$X_{15A} + X_{15B} + X_{15C} = 1.130{,}4$$
$$X_{16A} + X_{16B} + X_{16C} = 1.130{,}4$$
$$X_{17A} + X_{17B} + X_{17C} = 1.130{,}4$$
$$X_{18A} + X_{18B} + X_{18C} = 943{,}2$$
$$X_{19A} + X_{19B} + X_{19C} = 945$$
$$X_{20A} + X_{20B} + X_{20C} = 945$$
$$X_{21A} + X_{21B} + X_{21C} = 941{,}4$$
$$X_{22A} + X_{22B} + X_{22C} = 941{,}4$$
$$X_{23A} + X_{1B} + X_{1C} = 941{,}4$$
$$X_{24A} + X_{24B} + X_{24C} = 982{,}8$$
$$X_{25A} + X_{25B} + X_{25C} = 984{,}6$$
$$X_{26A} + X_{26B} + X_{26C} = 982{,}8$$
$$X_{27A} + X_{27B} + X_{27C} = 981$$
$$X_{28A} + X_{28B} + X_{28C} = 981$$
$$X_{29A} + X_{29B} + X_{29C} = 981$$
$$X_{30A} + X_{30B} + X_{30C} = 981$$

3.3 Unbalanced Transport Model

Table 6 Unbalanced Transport

RW \ BS	A	B	C	Supply(Kg)
RW 1 (DM) **[1]**	14	14	11	563,4
RW 2 (DM) **[2]**	16	16	11	565,2
RW 3 (DM) **[3]**	14	17	11	563,4
RW 4 (DM) **[4]**	14	14	12	565,2
RW 5 (DM) **[5]**	14	14	11	563,4
RW 6 (DM) **[6]**	13	13	11	561,6
RW 7 (DM) **[7]**	12	12	11	561,6
RW 8 (DM) **[8]**	13	13	11	561,6
RW 9 (DM) **[9]**	13	13	12	561,6
RW 1 (KT) **[10]**	8,8	8,8	9	1132,1
RW 2 (KT) **[11]**	8,8	8,8	8,8	1132,1
RW 3 (KT) **[12]**	5	5	8,8	1130,4
RW 4 (KT) **[13]**	8,8	8,8	10	1130,4
RW 5 (KT) **[14]**	8,8	9,2	5	1130,4
RW 6 (KT) **[15]**	8,8	8,8	9	1130,4
RW 7 (KT) **[16]**	8,9	8,8	8,9	1130,4
RW 8 (KT) **[17]**	8,8	8,8	8,8	1130,4
RW 1 (M) **[18]**	11	14	8,8	943,2
RW 2 (M) **[19]**	8,8	11	8,8	945
RW 3 (M) **[20]**	8,8	10	8,8	945
RW 4 (M) **[21]**	8,8	14	8,8	941,4
RW 5 (M) **[22]**	10	13	9,1	941,4
RW 6 (M) **[23]**	9,8	12	8,8	941,4
RW 1 (G) **[24]**	8,8	9,4	8,8	982,8
RW 2 (G) **[25]**	8,8	8,8	8,8	984,6
RW 3 (G) **[26]**	8,8	8,8	9	982,8
RW 4 (G) **[27]**	9,1	11	8,8	981
RW 5 (G) **[28]**	8,8	11	8,8	981
RW 6 (G) **[29]**	8,8	9,6	8,8	981
RW 7 (G) **[30]**	9,2	12	8,8	981
Demand (Kg)	8.200	8.500	8.600	25.300 \ 26.645,2

Because the supply is greater than demand, it is included in the category of unbalanced transportation model. So to solve it, you have to use a dummy variable, by increasing the demand by1,345.2 kg and the dummy column has a value of 0 (zero).

Table 7 Added Dummy Variable

BS / RW	A	B	C	Dummy	Supply (Kg)
[1]	14	14	11	0	563,4
[2]	16	16	11	0	565,2
[3]	14	17	11	0	563,4
[4]	14	14	12	0	565,2
[5]	14	14	11	0	563,4
[6]	13	13	11	0	561,6
[7]	12	12	11	0	561,6
[8]	13	13	11	0	561,6
[9]	13	13	12	0	561,6
[10]	8,8	8,8	9	0	1132,1
[11]	8,8	8,8	8,8	0	1132,1
[12]	5	5	8,8	0	1130,4
[13]	8,8	8,8	10	0	1130,4
[14]	8,8	9,2	5	0	1130,4
[15]	8,8	8,8	9	0	1130,4
[16]	8,9	8,8	8,9	0	1130,4
[17]	8,8	8,8	8,8	0	1130,4
[18]	11	14	8,8	0	943,2
[19]	8,8	11	8,8	0	945
[20]	8,8	10	8,8	0	945
[21]	8,8	14	8,8	0	941,4
[22]	10	13	9,1	0	941,4
[23]	9,8	12	8,8	0	941,4
[24]	8,8	9,4	8,8	0	982,8
[25]	8,8	8,8	8,8	0	984,6
[26]	8,8	8,8	9	0	982,8
[27]	9,1	11	8,8	0	981
[28]	8,8	11	8,8	0	981
[29]	8,8	9,6	8,8	0	981
[30]	9,2	12	8,8	0	981
Demand (Kg)	8200	8500	8600	1345,2	26.645,2 / 26.645,2

3.4 Result VAM Method

Table 8 Result VAM Method

BS / RW	A	B	C	Dummy	Supply (Kg)
1	0	0	345	218,4	563,4
2	0	0	565,2	0	565,2
3	0	0	563,4	0	563,4

4	0	0	0	565,2	565,2
5	0	0	563,4	0	563,4
6	0	0	561,6	0	561,6
7	0	0	561,6	0	561,6
8	0	0	561,6	0	561,6
9	0	0	0	561,6	561,6
10	253,2	878,9	0	0	1132,1
11	0	1132,1	0	0	1132,1
12	1130,4	0	0	0	1130,4
13	0	1130,4	0	0	1130,4
14	0	0	1130,4	0	1130,4
15	0	1130,4	0	0	1130,4
16	0	1130,4	0	0	1130,4
17	0	1130,4	0	0	1130,4
18	0	0	943,2	0	943,2
19	945	0	0	0	945
20	945	0	0	0	945
21	941,4	0	0	0	941,4
22	0	0	941,4	0	941,4
23	0	0	941,4	0	941,4
24	982,8	0	0	0	982,8
25	0	984,6	0	0	984,6
26	0	982,8	0	0	982,8
27	981	0	0	0	981
28	981	0	0	0	981
29	981	0	0	0	981
30	59,2	0	921,8	0	981
Demand (Kg)	8200	8500	8600	1345,2	26645,2

With, Z = 222837,3

3.5 MODI Calculation

When calculating the MODI index, there is no negative value, so the solution obtained from the initial feasible solution can be said to be the optimum solution with Z = 222837,3.To get the minimum cost multiplied by 1000 because the cost is in thousands so that we get Rp. 222,837,300.

4 Conclusion

In this study, we obtained mathematical modeling of the distribution management of plastic waste, mathematical modeling solutions for the management of plastic waste distribution, and produced an efficient plastic waste distribution system in the form of minimum costs for managing the distribution of plastic waste. So it can be concluded that the transportation method can be used to solve the problem of managing the distribution of plastic waste. By utilizing the transportation method, the minimum transportation cost is Rp. 222,837,300.

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