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Comparison of Double Exponential Smoothing and Double Moving Average for Forecasting Lost Vehicle Registration Certificates in Pamekasan

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Abstract. The government issues the Vehicle Registration Certificate (STNK), an official document that certifies a motorized vehicle's identity and authenticity. Pamekasan Regency is one of the regencies in East Java Province that frequently suffers losses associated with Vehicle Registration Certificates (STNK). Consequently, it is essential to predict the amount of car registration losses so that the Pamekasan regional administration can use the information to lower the losses. The Double Exponential Smoothing and Double Moving Average techniques were used in this study to forecast the amount of vehicle registration losses. According to the research findings, the smoothing parameters $\alpha = 0.3$ and $\gamma = 0.025$ had the lowest MAPE value from the Double Exponential Smoothing method, with a MAPE value of 49.4082%. The double moving average method's smallest MAPE, $k = 3$, has a MAPE value of 31.53215%. The twofold moving average approach is the best way to forecast the loss of car registration in Pamekasan, according to the comparison's findings.

Keywords: *double exponential smoothing; double moving average; loss of vehicle registration.*

1 Introduction

Loss is a term used to describe the state that a person experiences when they are cut off from something that was once theirs. Everyone has gone through loss at some point in their lives. People have suffered loss from birth and frequently do so again, albeit in various ways [1].

Vehicle Registration Certificate is shortened to STNK. This formal government-issued document serves as a motorized vehicle's identity and proof of legitimacy. Important details like the owner's name, owner identification number, car information, tax validity term, and vehicle registration number are all included in STNK. Every motorized vehicle is typically obliged to have a STNK, which the driver of the vehicle must carry at all times [2].

The loss of Vehicle Registration Certificates (STNK) is a major problem for the district of Pamekasan in the province of East Java. Of addition to causing administrative

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problems for car owners, losing STNK can be utilized for illegal operations like identity theft and the unlawful trading of vehicles. To lessen its detrimental effects, handling missing STNK calls for a clever and effective strategy [3]. In this situation, data analysis techniques like double moving average and double exponential smoothing may be useful in anticipating STNK loss in Pamekasan.

Two factors, level and trend, are taken into consideration when using the double exponential smoothing approach to forecast data trends [4]. Local governments can estimate the number of STNK losses in a certain time period by using this method to analyze STNK loss data. In order to address the issue of STNK loss, this can aid in more efficient resource allocation and policy planning.

By averaging two distinct time periods, the double moving average is an additional helpful technique for forecasting data trends [5]. Local governments can better comprehend the variations in the number of STNK losses over time by applying a double moving average to the STNK loss data in Pamekasan. By using this data, one can see certain trends that could influence the degree of STNK loss and take the necessary precautions.

The local administration of Pamekasan can estimate the number of STNK losses in the future more precisely by using these two techniques. As a result, they may be proactive in lowering the number of STNK losses and enhancing the effectiveness of motor vehicle administration management. Furthermore, more accurate STNK loss forecasting can help the community and other authorities better plan for any issues that might arise from misplaced vehicle documentation. Therefore, it is anticipated that the use of double exponential smoothing and double moving average in the examination of STNK loss data in Pamekasan will yield substantial advantages in successfully resolving this issue.

There are several researches about forecasting using double exponential smoothing or double moving average method. One of them is Manullang and Mansyur's study which is Forecasting Rice Sales at Perum Bulog Sub Divre Medan Using the Double Exponential Smoothing Method [6]. In this research, $\boldsymbol{\alpha = 0.2}$ with a Mean Absolute Percentage Error (MAPE) of 0.27% was the optimal α parameter for predicting the amount of rice sales at Perum Bulog Sub Divre Medan. Additionally, Perum Bulog Sub Divre Medan's 2022 rice sales prediction showed monthly declines.

Purwanto and Afiyah [4] carried out another study titled Corn Production Forecasting System in West Java Province Using the Double Exponential Smoothing Method. With the results of predicting corn output in West Java Province for the upcoming year of 1,049,839.84, the study's smallest MAPE was 9.38%, indicating an accuracy level of 90.62% at alpha 0.4.

Lumaksono et al. [7] carried out a second study titled Prediction of the Number of Legal Violations in Indonesian Seas Using the Double Exponential Smoothing Method. The study's findings indicate that, with a data smoothing constant value of $\alpha = 0.9$, a trend smoothing value of $\gamma = 0.2$, an average absolute percentage error value of MAPE $=$ 23.85%, and a Root Mean Square Error value (RMSE) of 67.726—the number of legal violations in Indonesian seas in 2020 will drop to 87 cases.

Irawan et al. did a second study titled Application of the Double Moving Average Method in Predicting Timber Demand [8]. According to this study, the DMA method's predictions are quite accurate. The demand for meranti wood for the upcoming period is 43 trunks with a MAPE value of 8%, and the demand for mahogany wood is 19 trunks with a MAPE value of 11%. In order to predict the number of foreign tourists arriving at Ngurah Rai Airport, a study named Comparison of Double Moving Average and Double Exponential Smoothing was carried out by Hudiyanti et al. [5]. According to testing conducted in 2017, the optimal time order value for Double Exponential Smoothing with parameter $\alpha = 0.4$ and Double Moving Average is 2. Based on these parameter values, the Double Moving Average's MAPE value is 10.522, and the Double Exponential Smoothing's MAPE value is 3.355. The quantity of international visitors arriving at Ngurah Rai Airport can therefore be predicted more accurately by Double Exponential Smoothing than by Double Moving Average.

Sinaga and Novica also did a study titled Comparison of Double Moving Average with Double Exponential Smoothing in Forecasting Medical Consumables [9]. Samples, specifically 3ml syringe data, were collected for this investigation between January 1 and June 30, 2017. Sinaga generated MAPE and RMSE from these data, demonstrating that the Double Moving Average Method outperforms the Double Exponential Smoothing Method in terms of accuracy (MAPE = 0.353 and RMSE = 95.8).

Listiowarni, et al [10] did a study titled Comparison of Double Exponential Smoothing and Double Moving Average Methods for Forecasting Retail Rice Prices in Pamekasan Regency. The Double Moving Average approach was shown to be superior based on the tests conducted, with an MSE value of 6349.25 and a MAPE value of 0.582542% utilizing a time order of 3. Using the DMA technique, the average monthly retail rice price projected for 2019 was Rp12.169. This study will compare the Double Exponential Smoothing and Double Moving Average approaches in order to anticipate the frequency of STNK losses in Pamekasan. To choose between the two approaches, the Average Percentage Error (MAPE) method is employed.

From the description above, the researcher examined the forecasting of STNK loss in Pamekasan by comparing Double Exponential Smoothing and Double Moving Average Methods.

2 Research Methods

2.1 Data Identification

In this study, the author uses a quantitative approach, where the author will work with numbers as a manifestation of the observed symptoms. The population in this study is the number of STNK R2 loss data in Pamekasan for one year, the sample taken is data on the number of losses per week in 2023. The data was obtained based on the number of loss reports covered by Radar Madura Pamekasan during 2023. There are 52 weeks data covered where 26 data used as insample data and 26 others used as outsample data.

Table 1 Number of vehicle registration losses in Pamekasan in 2023

Week	Data	Week	Data	Week	Data	Week	Data	Week	Data
1	12	12	10	23	20	34	15	45	12
2	24	13	12	24	22	35	14	46	9
3	19	14	23	25	25	36	14	47	10
4	12	15	11	26	2	37	12	48	6
5	23	16	4	27	16	38	16	49	11
6	18	17	9	28	11	39	8	50	14
7	17	18	21	29	14	40	16	51	11
8	10	19	25	30	8	41	13	52	5
9	21	20	20	31	12	42	19		
10	12	21	17	32	17	43	14		
11	15	22	14	33	15	44	16		

This data is used as material to predict the number of lost STNK in Pamekasan in the future using the Double Exponential Smoothing and Double Moving Average comparison methods.

2.2 Forecasting Using the Holt Two-Parameter Double Exponential Smoothing Method

Double Exponential Smoothing is a forecasting method used to forecast time series data with changing trends and patterns. This method is a development of the Single Exponential Smoothing (SES) method by adding parameters to estimate trends in the data. DES uses two parameters, namely alpha and beta. The purpose of using alpha is to calculate the level and beta to calculate the time series [11]. The formula for the double exponential smoothing (DES) Two Parameter method is as follows [12]:

$$S_t = \alpha X_t + (1 - \alpha)(S_{t-1} + b_{t-1}) \quad (1)$$

$$b_t = \gamma(S_t - S_{t-1}) + (1 - \gamma)b_{t-1} \quad (2)$$

$$F_{t+m} = S_t + b_t m \quad (3)$$

Where

S_t : forecast for period t.

X_t : value

b_t : trend in period t
α : first parameter of alignment
γ : the second parameter for trend smoothing is 0.025
F_{t+m} : forecast result t – m
m : number of periods ahead to be forecasted.

The initialization process for Holt's double exponential smoothing requires estimated data, namely determining the value of S_1, and determining the trend value b_1. For the first one, select $s_1 = x_1$, for the estimated trend value b_1 requires estimates from one period to another [12]. One of them is by using:

$$b_1 = \frac{\big((x_2 - x_1) + (x_3 - x_2\big) + (x_4 - x_3)}{3}$$

$$b_1 = \frac{((24 - 12) + (19 - 24) + (12 - 19))}{3}$$

$$b_1 = 0$$

Once the estimated values for s_1 and b_1 are known, the forecast for the next period uses the double exponential smoothing formula (equations (1) to (3)).

The forecast for period 2 is:

$$\begin{aligned} F_2 &= S_1 + b_1(1) \\ &= 12 + (0)(1) \\ &= 12 \end{aligned}$$

Forecasting calculations use different alpha values, namely 0.3, 0.5, and 0.7. From the experiment, the smallest MAPE value is at alpha 0.3. The following is the calculation of the alpha value of 0.3.

For forecasting in period 3, namely:

$$\begin{aligned} S_2 &= \alpha X_2 + (1 - \alpha)(S_1 + b_1) \\ &= 0{,}3(24) + (1 - 0{,}3)(12 + 0) \\ &= 15{,}6 \\ b_2 &= \gamma(S_2 - S_1) + (1 - \gamma)b_1 \\ &= 0{,}025(15{,}6 - 12) + (1 - 0{,}025) \times 0 \\ &= 0{,}09 \\ F_3 &= S_2 + b_2(1) \\ &= 15{,}6 + 0{,}09(1) \\ &= 15{,}69 \end{aligned}$$

2.3 Forecasting Using the Double Moving Average Method

Double moving average is a moving average model that uses a number of new actual demand data to generate forecast values in the future. Double moving average is the result of two calculations from the single moving average (SMA) method [11]. The steps in determining the Double Moving Average forecast method are as follows [5].

$$S'_t = \frac{x_t + x_{t-1} + x_{t-2} + \cdots + x_{t-k+1}}{k} \quad (4)$$

$$S''_t = \frac{S'_t + S'_{t-1} + \cdots + S'_{t-k+1}}{k} \quad (5)$$

$$a_t = 2S'_t - S''_t \quad (6)$$

$$b_t = \frac{2}{k-1}(S'_t - S''_t) \quad (7)$$

$$F_t = a_t + b_t(m) \quad (8)$$

Where

F_{t+m} : forecast on period $(t + m)$
a_t : adjusted average value at period t or constant value
b_t : trend coefficient at period t
x_t : actual data value at period t
S'_t : first moving average in period t
S''_t : second moving average at period t
m : future time period
k : time order used

Forecasting calculation using Double Moving Average with $k = 3$

$$S'_t = \frac{x_t + x_{t-1} + x_{t-2} + \cdots + x_{t-k+1}}{k}$$

$$S'_3 = \frac{x_3 + x_2 + x_1}{3}$$

$$= \frac{19 + 24 + 12}{3}$$

$$= 18{,}33333$$

$$S''_t = \frac{S'_t + S'_{t-1} + \cdots + S'_{t-k+1}}{k},$$

$$S''_5 = \frac{S'_5 + S'_4 + S'_3}{k}$$

$$= \frac{18{,}33333 + 18{,}33333 + 18}{3}$$

$$= 18{,}22222$$

Determine the magnitude of a_t by substituting equation (6)

$$a_t = 2S'_t - S''_t$$
$$a_5 = 2S'_5 - S''_5$$
$$= 2(18) - 18{,}22222$$
$$= 36 - 18{,}22222$$
$$= 17{,}77778$$

Determine the magnitude of b_t by substituting equation (7)

$$b_t = \frac{2}{k-1}(S'_t - S''_t)$$
$$b_5 = \frac{2}{3-1}(S'_5 - S''_5) = \frac{2}{2}(S'_5 - S''_5)$$
$$b_5 = \frac{2}{2}(18 - 18{,}2222)$$
$$= -0{,}2222$$

Determine the magnitude of the forecast value by substituting equation (8)

$$F_t = a_t + b_t(1)$$
$$F_5 = a_5 + b_5(1)$$
$$= 17{,}77778 + (-0{,}2222)(1)$$
$$= 17{,}55556$$

Forecasting calculation using Double Moving Average with $k = 5$

$$S'_t = \frac{x_t + x_{t-1} + x_{t-2} + \cdots + x_{t-k+1}}{k}$$
$$S'_5 = \frac{x_5 + x_4 + x_3 + x_2 + x_1}{5}$$
$$= \frac{23 + 12 + 19 + 24 + 12}{5}$$
$$= 18$$
$$S''_t = \frac{S'_t + S'_{t-1} + \cdots + S'_{t-k+1}}{k}$$
$$S''_9 = \frac{S'_9 + S'_8 + S'_7 + S'_6 + S'_5}{5}$$
$$= \frac{17{,}8 + 16 + 17{,}8 + 19{,}2 + 18}{5}$$
$$= 17{,}76$$

Determine the magnitude of a_t by substituting equation (6)

$$a_t = 2S'_t - S''_t$$
$$a_9 = 2S'_9 - S''_9$$

$$= 2 \times 17{,}8 - 17{,}76 = 17{,}84$$

Determine the magnitude of b_t by substituting equation (7)

$$b_t = \frac{2}{k-1}(S'_t - S''_t),$$

$$b_9 = \frac{2}{5-1}(S'_9 - S''_9) = \frac{1}{2}(S'_9 - S''_9)$$

$$= \frac{1}{2} \times 17{,}8 - 17{,}76$$

$$= 0.02$$

Determine the magnitude of the forecast value by substituting equation (8)

$$F_t = a_t + b_t(1)$$

$$F_9 = a_9 + b_9(1)$$

$$= 17{,}84 + 0{,}02$$

$$= 17{,}86$$

Forecasting calculation using Double Moving Average with $k = 5$

$$S'_t = \frac{x_t + x_{t-1} + x_{t-2} + \cdots + x_{t-k+1}}{k}$$

$$S'_7 = \frac{x_7 + x_6 + x_5 + x_4 + x_3 + x_2 + x_1}{7}$$

$$= \frac{17 + 18 + 23 + 12 + 19 + 24 + 12}{7}$$

$$= 17{,}85714$$

$$S''_t = \frac{S'_t + S'_{t-1} + \cdots + S'_{t-k+1}}{k}$$

$$S''_{13} = \frac{S'_{13} + S'_{12} + S'_{11} + S'_{10} + S'_9 + S'_8 + S'_7}{7}$$

$$= \frac{13{,}85714+14{,}71429+16{,}57143+16{,}14286+17{,}14286+17{,}57143+17{,}85714}{7}$$

$$= 16{,}26531$$

Determine the magnitude of a_t by entering equation (6)

$$a_t = 2S'_t - S''_t$$

$$a_{13} = 2S'_{13} - S''_{13}$$

$$= 2 \times 13{,}85714 - 16{,}26531$$

$$= 11{,}44898$$

Determine the magnitude of b_t by entering equation (7)

$$b_t = \frac{2}{k-1}(S'_t - S''_t)$$

$$b_{13} = \frac{2}{7-1}(S'_{13} - S''_{13}) = \frac{1}{3}(S'_{13} - S''_{13})$$

$$= \frac{1}{3}(13{,}85714 - 16{,}26531) = -0{,}80272$$

Determine the magnitude of the forecast value by entering equation (8)

$$F_t = a_t + b_t(1)$$

$$F_{13} = a_{13} + b_{13}(1)$$

$$= 11{,}44898 + (-0{,}80272)$$

$$= 10{,}64626$$

2.4 Evaluation of Forecasting Results Using MAPE for the Double Exponential Smoothing and Double Moving Average Methods

Mean Absolute Percentage Error (MAPE) is a calculation of the difference between actual data and forecasted data that has been absolute, with the aim of avoiding minus (negative) values and then calculated in the form of a percentage of the original data. A method is said to have good performance if it produces a MAPE value of less than 10%, or it can be said that the smaller the MAPE value, the better the performance of a method being measured, because MAPE can be used as an evaluation calculation, how precise and accurate a prediction is produced by a particular method [10].

Table 2 MAPE value criteria

MAPE Value	**Criteria**
MAPE < 10%	Very good
10% ≤ MAPE ≤ 20%	Good
20% ≤ MAPE ≤ 50%	Worthy
MAPE > 50%	Not feasible

Equation (9) shows the calculation formula to obtain the MAPE (Mean Absolute Percentage Error) value.

$$\text{MAPE} = \frac{\sum_{t=1}^{n} \left| \left(\frac{x_i - f_i}{x_i} \right) \right|}{n} \times 100\%$$

Where

x_i : Data for the $t - th$ period

f_i : $t - th$ forecasting data

n : number of forecast periods.

3 Results and Discussion

Forecasting results using the Double Exponential Smoothing method on STNK loss data in Pamekasan by comparing the alpha values used. The difference in alpha values will produce different S'' and b_t values. The results of several different alpha value experiments will produce different error values (MAPE). Comparison of MAPE values in forecasting using the Double Exponential Smoothing method as follows:

Table 3 Comparison of MAPE values in the double exponential smoothing method

	Alpha = 0.3	**Alpha = 0.5**	**Alpha = 0.7**
MAPE	49.40816%	50.48282%	50.05021%

The following is the plot result between actual data and forecasted data using the Double Exponential Smoothing method, which can be seen in Figure 1.

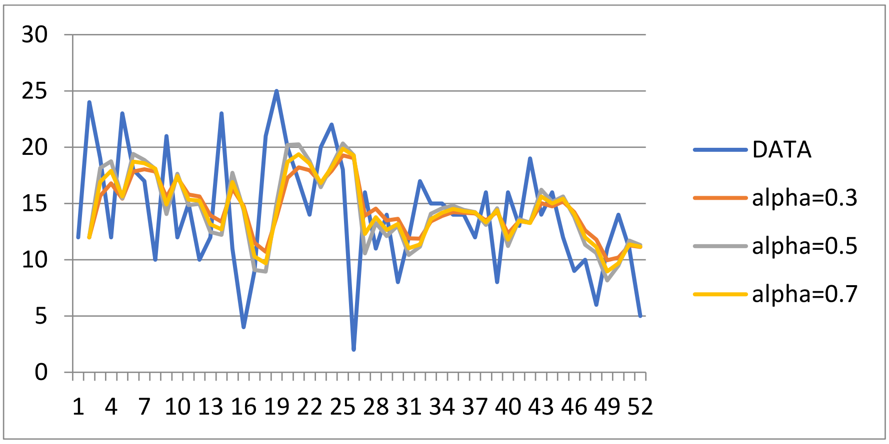

Figure 1 Graph of actual data and forecast results

The results of forecasting using the Double Moving Average method on STNK loss data in Pamekasan by comparing the values of the many periods used. The difference in k values will produce different S'' and b_t values. The results of several trials of different k values will produce different error values (MAPE). Comparison of MAPE values in forecasting using the Double Moving Average method as follows:

Table 4 Comparison of MAPE values in the double moving average method

	$\boldsymbol{k = 3}$	$\boldsymbol{k = 5}$	$\boldsymbol{k = 7}$
MAPE	31.53215%	33.42264%	40.55431%

The following is the plot result between actual data and forecast data using the Double Moving Average method, which can be seen in Figure 2.

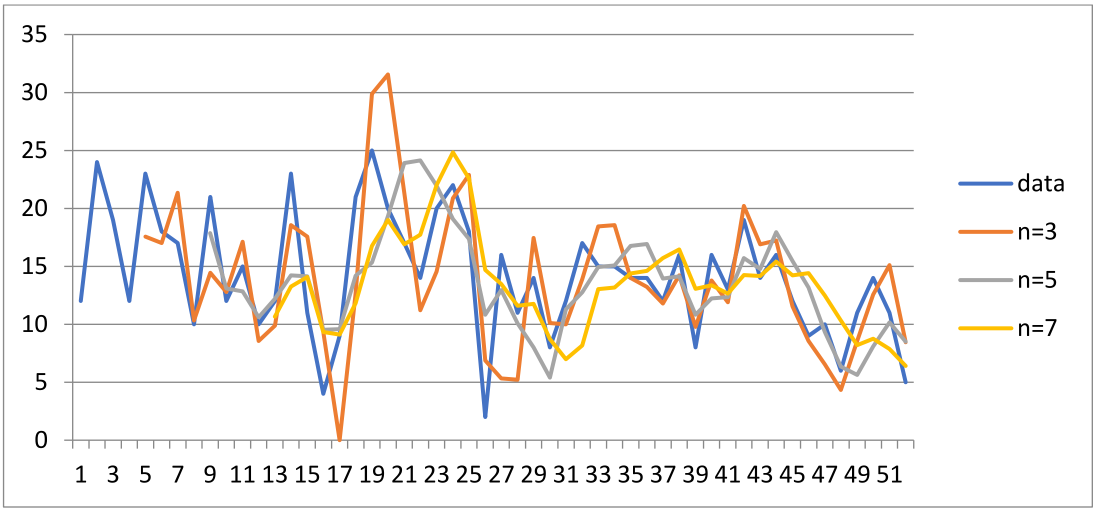

Figure 2 Graph of actual data and forecast results

The following are the plot results between actual data and comparison data using the Double Exponential Smoothing and Double Moving Average methods, which can be seen in Figure 3.

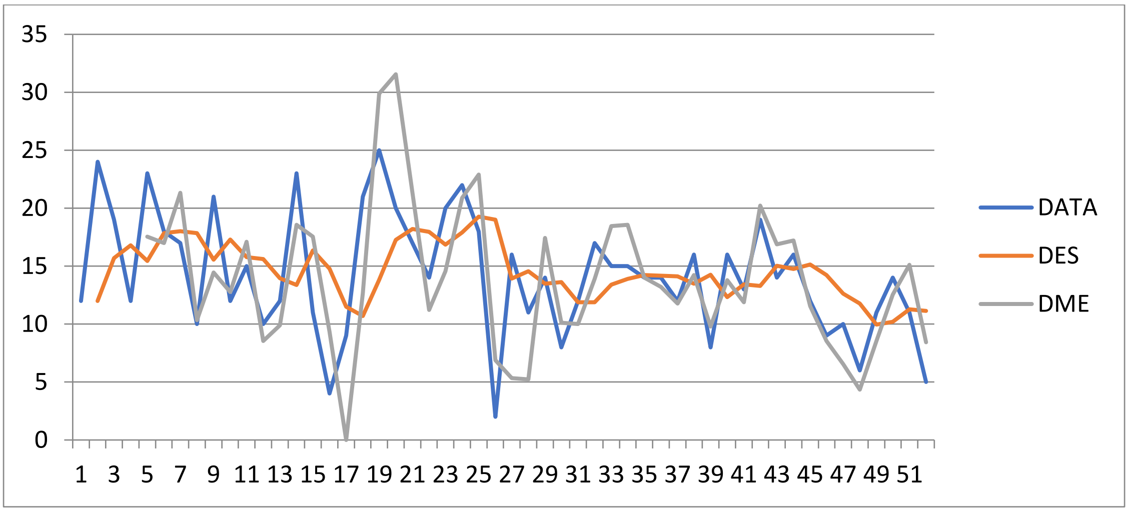

Figure 3 Actual data graph and comparison results of DES method and DMA method

4 Conclusion

From the discussion above, the smallest MAPE of the double exponential smoothing method found in the smoothing parameters $a = 0.3$ and $\gamma = 0.025$ is 49.4082%. While the best double moving average method is found at $k = 3$ which is 31.53215%. Both methods have decent performance because the MAPE value is between $20\% - 50\%$, but the Double Moving Average method has a smaller MAPE value compared to the Double

Exponential Smoothing method. From the comparison of the two methods above, it can be seen that the one that can be used to predict the loss of STNK in Pamekasan is the Double Moving Average method with a MAPE value of 31.53215%.

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